4.1 & 4.2 Prime Numbers and GCF

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ number is a number whose only factors are 1 and that number.

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ number has factors other than 1 and itself.

Examples: 13 is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because the only factors are 1 and 13

 12 is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ because its factors are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What is the only even prime number? \_\_\_\_\_\_\_\_\_\_\_\_

What number is neither prime nor composite? \_\_\_\_\_\_\_\_\_\_\_\_\_

To find LCM –Least Common Multiple (which is the same as LCD – Least Common Denominator) or the GCF- Greatest Common Factor, the first thing you do is find the prime factorization using factor trees.

Find the GCF for 24 and 18

 24 18

Place the factors that BOTH numbers share in the shared part of the Venn Diagram (for this example both have one 2 and one 3) and mark them off of your prime factorization.

Place the rest of the factors in each oval so that they are all used (for this example, there are two 2’s left for the 24 and a 3 left for the 18).

The GCF is the product of the number in the shared part of the Venn Diagram (2 x 3 = 6 in our example).

REMEMBER GCF IS THE PRODUCT OF THE SHARED FACTORS!

Find GCF for:

1. 18 and 27 2. 63 and 84

GCF: \_\_\_\_\_\_\_ GCF: \_\_\_\_\_\_\_\_

3. 154 and 462 4. 12, 78, and 96

 GCF: \_\_\_\_\_\_\_ GCF: \_\_\_\_\_\_\_

You do the SAME if it is a monomial. A monomial is an algebraic expression with only ONE term.

Monomial factored: 8x2 = $2∙2∙2∙x∙x$

7. 8x2 and 6x 8. 9x3 and 12x2y

GCF: \_\_\_\_\_\_ GCF: \_\_\_\_\_\_\_\_

9. 14x and 21xy2 10. 12x2y4 and 18xy3

GCF: \_\_\_\_\_\_\_ GCF: \_\_\_\_\_\_\_\_\_

11. 7x and 9y

GCF: \_\_\_\_\_\_\_\_

Note that 7x and 9y are relatively prime – they have no common factors (but 1)

HW: 4.1 (page 177) #34 – 44 even, 46, 48-54 even

AND 4.2 (page 181) #12 – 30 even, 32 (hint use GCF), 38-46 even

4.3 Equivalent Fractions

Equivalent fractions are two fractions that represent the same number (example $\frac{1}{2}$ and$ \frac{2}{4}$). We will need common denominators in order to add or subtract fractions.

Sometimes we need to reduce a fraction that is not in lowest terms. We know that a fraction is not in lowest terms if the numerator and denominator have a common factor other than 1.

Example:  🡨 2 and 3 will both go into 18 and 24 so 6 is the GCF.

Divide the top and bottom by the GCF: 

1.  2. 

3.  4. 

If the numerator is larger than the denominator, then you have an improper fraction and usually these need to be turned into mixed numbers.

 divide 7 into 18 you get 2 remainder 4 so the answer is: 

5.  6.  7. 

If the numerator and denominator are monomials, remember to factor the variables and cross out any common factors.

Examples $\frac{4x^{2}}{6x^{5}}$ =

8. $\frac{12x^{5}}{27x^{3}}$

9. $\frac{15x^{6}y^{2}}{35x^{3}y^{5}}$

10. $\frac{20xy^{7}}{21y}$

11. $\frac{18xyz}{45x^{2}y^{3}z}$

HW: 4.3 (page 187) #16 – 52 even

4.4 Least Common Multiple

A common denominator is needed to add and subtract fractions. To find the common denominator, you find the LCM (least common multiple) also known as least common denominator (LCD).

Find the LCM for 24 and 18

 24 18

LCM = \_\_\_\_\_\_\_\_\_\_\_\_\_\_× \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ × \_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_

 (from the left) (from the middle) (from the right)

The LCM (Least Common Multiple), also known as the LCD (Least Common Denominator) is the product of ALL numbers in the Venn Diagram.

As before, the GCF is the product of the number in the shared part of the Venn Diagram.

(Review) GCF = \_\_\_\_\_\_\_\_\_\_\_\_\_

You try finding LCD and GCF:

1. 18 and 27 2. 12 and 9

GCF: \_\_\_\_\_\_\_ GCF: \_\_\_\_\_\_\_

LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_

3. 10 and 25 4. 12, 78, 96

GCF: \_\_\_\_\_\_ GCF: \_\_\_\_\_\_

LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_

5. 8x2 and 6x 6. 9x3 and 12x2y

GCF: \_\_\_\_\_\_\_ GCF: \_\_\_\_\_\_\_

LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_

7. 14x and 21xy2 8. 12x2y4 and 18xy3

GCF: \_\_\_\_\_\_\_ GCF: \_\_\_\_\_\_\_

LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ LCM: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_

HW: 4.4 (page 192) #16-34 even, 40-54 even

4.5 Rules of Exponents

Part 1: Multiplication.

Simplify: $x^{2}\left(x^{5}\right)=$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_

 $7^{2}∙7^{8}$ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_

When **multiplying** exponents WITH LIKE BASES: \_\_\_\_\_\_\_\_\_\_\_the exponents.

* Do NOT multiply numerical bases!!!

If the bases are different, you cannot simplify: *x*2 *y*3 can’t be simplified.

Simplify: $2x^{2}y^{2}∙4x^{3}y^{5}=$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_

* You must multiply coefficients.

Practice. Find the product. Write your answer using exponents.

1. $x^{2}x^{7}=$ 2. $3^{5}∙3^{6}=$

3. $2x^{4}x^{5}=$4. $7x^{2}y^{6}∙4x^{5}y^{3}=$

5. $5xy^{3}∙5xy^{2}=$ 6. $2x^{4}y^{9}\left(-5xy^{2}\right)= $

Part 2: Division:

Simplify: $\frac{x^{5}}{x^{2}}=$

 $\frac{11^{7}}{11^{4}}=$

When **dividing** exponents WITH LIKE BASES: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the exponents.

* All answers should have positive exponents.
* DO NOT DIVIDE NUMERICAL BASES!

REMEMBER: Anything to the zero power is \_\_\_\_\_\_\_\_\_. Here’s why:

Simplify: $\frac{y^{3}}{y^{3}}$ =

Make sure you reduce all fractions including the constants. If everything cancels you get ONE (not zero).

Simplify: $\frac{4x^{4}}{18x^{2}}$ =

Simplify: $\frac{36x^{9}y^{5}}{12x^{4}y^{2}}$ =

Practice. Find the quotient. Write your answer using exponents.

7. $\frac{9^{10}}{9^{4}}=$ 8. $\frac{x^{4}y^{9}}{xy^{3}}$ =

9. = 10. 

HW: 4.5 (page 198) # 16 – 38 even, 42-56 even, 59-63 all

4.6 Negative Exponents

Complete the table:

|  |  |  |
| --- | --- | --- |
| Exponential Form | Expanded Form | Value |
| 104 | $10 × 10 × 10 × 10$  |  |
| 103 | $$10 × 10 × 10$$ |  |
| 102 | $$10 × 10$$ |  |
| 101 | $$10 $$ |  |
| 100 |  |  |
| 10-1 |  |  |
| 10-2 |  |  |
| 10-3 |  |  |
| 10-4 |  |  |

When working with negative exponents, we can rewrite the value with a positive exponent by moving the value to the denominator.

If a negative exponent appears in the denominator, move it to the numerator.

Write the expressions using only positive exponents.

1. $7^{-3} $= 2. $8^{-5}$ = 3. $95^{0} $=

This rule applies to algebraic expressions as well.

4. $3x^{0}y^{-8}$ = 5. $m^{4}n^{-2}$= 6. $a^{-2}b^{-3}$=

We can also start with fractions and change them to expressions with negative exponents. Use the prime factorization to express these values.

Write an equivalent expression without a fraction bar.

7. $\frac{1}{16}$ = 8. $\frac{1}{49} $= 9. $\frac{1}{1000}$ =

10. $\frac{1}{m^{4}n^{2}}$ = 11. $\frac{2a^{3}}{b^{3}}$ = 12. $\frac{x^{2}}{y^{5}z^{-7}}$ =

* We can multiply and divide exponential expressions when the bases are the same.

To find the product: Add the exponents.

To find the quotient: Subtract the exponents. Remember your integer rules!

$$3^{8}∙3^{-4}=$$

$$\frac{2.1x^{-3}}{3x^{4}}=$$

Find the product or quotient. Write your answer using only positive exponents.

13. $10^{-3}∙ 10^{10}=$ 14. $(0.3)^{-9} ∙(0.3)^{-4}= $

14. $p^{2}∙p^{-5}=$ 15. $3x^{-8} ∙ -5x$ =

16. $\frac{6^{3}}{6^{5}}=$ 17. $\frac{y^{-5}}{y^{-2}}=$

18. $\frac{35k^{3}}{-5k^{-8}}=$ 19. $\frac{7x^{2}y^{3}}{7xy^{8}}=$

HW: 4.6 (page 204) #16-30 even, 32-39 all, 43-50 all, 56-59

4.7 Multiplying and Dividing with Scientific Notation

Review:

Mathematicians and scientists use scientific notation to simplify the numbers that they write. In chemistry for example, 600,000,000,000,000,000,000,000 is a mole but we never write it like that.

Instead we write $6 × 10 $

$6 × 10^{5}=6 × 100,000 =$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The easiest way to think about it is that we move the decimal point 5 places to the RIGHT when the exponent is **positive** (remember that positive is to the right on the number line).

5 × 10-4 = 5 × .0001 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The easiest way to think about it is that we move the decimal point 4 places to the LEFT when the exponent is **negative** (remember that negative is to the left on the number line).

Practice:

1. 5.23 × 106 2. 3.1 × 10-3

3. 9.789 × 108 4. 6.12 × 10-6

We can also take a number in Standard Notation and put it in Scientific Notation.

Example 1: 4,100,000 🡪 first we figure out where the decimal will go by making a number between 1 and 9.99999.

In this case our first factor will be \_\_\_\_\_\_\_\_\_\_\_\_.

We must move \_\_\_\_\_\_\_ places to the RIGHT. The exponent will be \_\_\_\_\_\_\_\_\_.

4,100,000 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example 2: 0.00087 🡪our first factor will become \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

We must move the decimal \_\_\_\_\_\_\_\_\_\_ places LEFT. The exponent is \_\_\_\_\_\_\_\_\_\_.

0.00087 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

5. 56,000,000 6. 0.0000213 7. 42,120,000,000

Multiplying and Dividing:

It is convenient to use scientific notation when multiplying or dividing large and small numbers:

(2.43 × 104)(7.54 × 10-2) = (2.43 × 7.54)(104 × 10-2) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

This is not proper scientific notation, so we must change it. 18.322 needs to be changed to 1.8322 which means we divided by \_\_\_\_\_\_\_\_\_\_\_. That means we must multiply the 102 by 10 for it to be equal.

\*In other words: If we need to make the first factor smaller, we make the second factor larger.

18.322 × 102 = (18.322/10) × (102 × 10) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The same method is used for dividing:

$\frac{2.1 × 10^{3}}{7 × 10^{5}}$ =

This is not proper scientific notation, so we must change it. 0.3 must be changed to 3.0. This means we multiplied it by \_\_\_\_\_\_\_\_\_. That means we must \_\_\_\_\_\_\_\_\_\_\_\_ 10-2 by 10.

\*In other words: If we need to make the first factor larger, we make the second factor smaller.

(0.3 × 10) × (10-2/10) = ­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Try these:

1. (2 × 102) × (3 × 104) 2. (8 × 10-4) × (4 × 103)

3. 4.2 x 10-3 4. 8.4 x 107

 7 x 10-7 2.1 x 10-2

HW: 4.7 (page 209) #14 – 50 even, Worksheet (You may use calculator to find the first factor, if needed.)

Scientific Notation Worksheet

Do these problems using scientific notation. **Your final answer should be in proper final scientific notation.** If the calculator answer is not in proper scientific notation form, you will have to change it. The decimal portion of your answer should be rounded to the nearest tenth.

1. (2 x 102)(3 x 104) 2. 8 x 10-4

 4 x 10 3

3. (2.3 x 106)(4.2 x 10-11) 4. (6.5 x 103)(5.2 x 10-8)

5. (2.34 x 10-8)(5.7 x 10-4) 6. (3.26 x 10-6)(8.2 x 10-6)

7. 8.5 x 108 8. 5.1 x 106

 3.4x105 3.4 x 103

9. 4.0 x 10-6 10. 7.5 x 10-9

 8.0 x 10-3 2.5 x 10-4

4.7- Day 2: Adding and Subtracting Scientific Notation

When adding or subtracting numbers in Scientific Notation, we must remember that **place value matters.**

This means, in order to add or subtract, our place values must be lined up. To do this, we must make sure that our powers of ten match for both numbers.

Example 1: Addition

$$\left(2.87 × 10^{4}\right)+(3.912 × 10^{3})$$

* We can change $3.912 × 10^{3}$ by changing the second factor to \_\_\_\_\_\_\_\_\_\_.
* This means we made our second factor larger (multiplied by 10), so our first factor must be \_\_\_\_\_\_\_\_\_\_ (divide by 10). This keeps the value “balanced.” (The product stays the same.)
* When both values have 10 to the same power, we simply line up the decimals and add or subtract.
* Always check that your final answer is in correct scientific notation (the first factor is a value between 1-10.)
* We can also change the first value, $2.87 × 10^{4}$, so that the second factor is $10^{3}$, but we would then need to change our final answer so that it is in true scientific notation.
* I usually like to change the smaller exponent to match the larger exponent.

Example 2: Subtraction

$$(3.21 × 10^{6})-(8.47 × 10^{4})$$

Example 3: Negative Exponents

$$(2.15 × 10^{-10})+(6.3 ×10^{-8})$$

* We are going to use the same technique that we did for positive exponents: Change the smaller exponent to match the larger exponent, and then add the decimals.
* Remember to keep each value “balanced.”

Practice:

1. $\left(3.14 × 10^{5}\right)+\left(8.02 × 10^{4}\right)$
2. $\left(7 × 10^{10}\right)-\left(9.2 × 10^{8}\right)$
3. $\left(4.09 × 10^{-3}\right)+\left(6 × 10^{-4}\right)$
4. $\left(4 × 10^{-2}\right)-\left(5 × 10^{-4}\right)$
5. $\left(1.36 × 10^{8}\right)+\left(2.9 × 10^{5}\right)$

HW: #1-8 on the next TWO pages